A Lattice Study of Spectator Effects in Inclusive Decays of B-Mesons

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Abstract

We compute the matrix elements of the operators which contribute to spectator effects in inclusive decays of B-mesons. The results agree well with estimates based on the vacuum saturation (factorization) hypothesis. For the ratio of lifetimes of charged and neutral mesons we find $\tau(B^-)/\tau(B_d) = 1.03 \pm 0.02 \pm 0.03$, where the first error represents the uncertainty in our evaluation of the matrix elements, and the second is an estimate of the uncertainty due to the fact that the Wilson coefficient functions have only been evaluated at tree-level in perturbation theory. This result is in agreement with the experimental measurement. We also discuss the implications of our results for the semileptonic branching ratio and the charm yield.

1   Introduction

Inclusive decays of heavy hadrons can be studied in the framework of the heavy quark expansion, in which widths and lifetimes are computed as series in inverse powers of the mass of the b-quark [1, 2, 3] (for recent reviews and additional references see refs. [4, 5]). The leading term of this expansion corresponds to the decay of a free-quark and is universal, contributing equally to the lifetimes of all beauty hadrons. Remarkably there are no corrections of

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$O(1/m_b)$, and the first corrections are of $O(1/m_b^2)$ [6, 3]. "Spectator effects", i.e. contributions from decays in which a light constituent quark also participates in the weak process, enter at third order in the heavy-quark expansion, i.e. at $O(1/m_b^3)$. However, as a result of the enhancement of the phase space for 2→2 body reactions, relative to 1→3 body decays, the spectator effects are likely to be larger than estimates based purely on power counting, and may well be significant. The need to evaluate the spectator effects is reinforced by the striking discrepancy between the experimental result for the ratio of lifetimes [7]:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.78 \pm 0.04 \ , \tag{1}$$

and the theoretical prediction

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.98 + O(1/m_b^3) \ . \tag{2}$$

In order to explain this discrepancy in the conventional approach, the higher order terms in the heavy quark expansion, and the spectator effects in particular, would have to be surprisingly large. In this paper we compute the matrix elements of the four-quark operators needed to evaluate these spectator effects for mesons; a computation of the effects for the Λ_b is in progress and the results will be reported in a future publication.

The experimental value of the ratio of lifetimes of the charged and neutral B-mesons is [7]

$$\frac{\tau(B^-)}{\tau(B_d^0)} = 1.06 \pm 0.04 \ , \tag{3}$$

to be compared to the theoretical prediction

$$\frac{\tau(B^-)}{\tau(B_d^0)} = 1 + O(1/m_b^3) \ . \tag{4}$$

Below we determine the contribution to the $O(1/m_b^3)$ term on the right-hand side of eq.(4) coming from spectator effects, which we believe to be the largest component. The same spectator effects also contribute to the semileptonic branching ratio of the B-mesons, and to the charm-yield (n_c, the average number of charmed particles in decays of B-mesons). This will be briefly discussed in section 5.

At $O(1/m_b^3)$, the non-perturbative contributions to the spectator effects are contained in the matrix elements of the four-quark operators:

$$
\begin{align}
O_{V-A}^q &= \bar{b}_L \gamma_\mu q_L \, \bar{q}_L \gamma^\mu b_L \tag{5}\\
O_{S-P}^q &= \bar{b}_R q_L \, \bar{q}_L b_R \tag{6}\\
T_{V-A}^q &= \bar{b}_L \gamma_\mu T^a q_L \, \bar{q}_L \gamma^\mu T^a b_L \tag{7}\\
T_{S-P}^q &= \bar{b}_R T^a q_L \, \bar{q}_L T^a b_R \; , \tag{8}
\end{align}
$$

where q represents the field of the light quark, and the T^a's are the generators of the colour group. Throughout this paper we take these operators to be defined at a renormalization scale m_b. The subscripts L and R represent "left" and "right" respectively. For mesons, following ref. [9], we introduce the parameters $B_{1,2}$ and $\varepsilon_{1,2}$:

$$
\frac{1}{2m_{B_q}} \langle B_q | O_{V-A}^q | B_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} B_1 \; , \tag{9}
$$

$$
\frac{1}{2m_{B_q}} \langle B_q | O_{S-P}^q | B_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} B_2 \; , \tag{10}
$$

$$
\frac{1}{2m_{B_q}} \langle B_q | T_{V-A}^q | B_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} \varepsilon_1 \; , \tag{11}
$$

$$
\frac{1}{2m_{B_q}} \langle B_q | T_{S-P}^q | B_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} \varepsilon_2 \; , \tag{12}
$$

where f_{B_q} is the leptonic decay constant of the meson B_q (in a normalization in which $f_\pi \simeq 131$ MeV). The introduction of the parameters $B_{1,2}$ and $\varepsilon_{1,2}$ is motivated by the vacuum saturation (or factorization) approximation [10] in which the matrix elements of four-quark operators are evaluated by inserting the vacuum inside the current products. This leads to $B_i = 1$ and $\epsilon_i = 0$ at some renormalization scale, μ, for the operators. It may be argued that the scale at which the factorization approximation holds is different from our chosen renormalization scale m_b, specifically that if the approximation is valid at all then it should hold at a typical hadronic scale of about 1 GeV [11]. In QCD, in the limit of a large number of colours N_c,

$$
B_i = O(1) \quad \text{and} \quad \varepsilon_i = O(1/N_c) \; . \tag{13}
$$

In this paper we evaluate the parameters B_i and ε_i in lattice simulations, thus testing the validity of the factorization hypothesis.

The results of our calculations indicate that the vacuum saturation hypothesis is (surprisingly?) well satisfied. In the $\overline{MS}$ renormalization scheme we find:

$$B_1(m_b) = 1.06(8) \qquad B_2(m_b) = 1.01(6) \qquad (14)$$

$$\varepsilon_1(m_b) = -0.01(3) \qquad \varepsilon_2(m_b) = -0.01(2) \qquad (15)$$

and

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.03 \pm 0.02 \pm 0.03 \ , \qquad (16)$$

in good agreement with the experimental value in eq. (3).

The present calculation is very similar to that of the B_B-parameter of B-$\bar{B}$ mixing, for which several recent simulations have been performed [12, 13, 14], including one using the same configurations used in this study [12]. We use the calculations of the B_B parameter as a comparison and check on our results, both for the perturbative matching coefficients and for the evaluation of the matrix elements. In performing this comparison we believe that we have found an error in the perturbative calculation. We also stress that the feature that the values of the matrix elements of the operators (5)-(8) are close to those expected from the vacuum saturation hypothesis is also present in the evaluation of the B_B-parameter.

The plan of the remainder of this paper is as follows. In the next section we derive the relation, at one-loop order of perturbation theory, between the operators defined in eqs. (5)-(8) and the bare lattice operators whose matrix elements are computed directly in lattice simulations. The technical details of the calculation are relegated to appendix A. In section 3 we present a description of the lattice computation, the results of this computation and a discussion of the implications. The comparison of our study to that of B^0-$\bar{B}^0$ mixing is presented in section 4 (the discussion of the evaluation of the matching coefficients for B-$\bar{B}$ mixing is described in detail in appendix B). Finally, in section 5 we present our conclusions.

2 Perturbative Matching

The Wilson coefficient functions of the operators (5)–(8) in the OPE for inclusive decay rates have been evaluated only at tree-level [9]. At this level of

precision it is sufficient to compute the matrix elements in any "reasonable" renormalization scheme. In this paper we present the results for the matrix elements of the HQET operators in the $\overline{MS}$ scheme at a renormalization scale m_b [1]. In order to obtain these from the matrix elements of bare operators in the lattice theory, with cut-off a^{-1} (where a is the lattice spacing), which we compute directly in our simulations, we require the corresponding matching coefficients. In this section we present these matching coefficients (at one-loop order in perturbation theory); we postpone a detailed description of their evaluation to Appendix A. In the following section we will use these coefficients and the computed values of the matrix elements of the bare operators to determine the coefficients B_i and ε_i in the $\overline{MS}$ scheme.

The coefficients presented in the section were all obtained using local lattice operators defined in the tree-level improved SW-action [15] defined in eq. (21) below.

It is convenient to perform the matching in two steps:

i) The first step is the evaluation of the coefficients which relate the HQET operators in the continuum ($\overline{MS}$) and lattice schemes, both defined at the scale a^{-1},

$$O_i^C(a^{-1}) = O_i^L(a^{-1}) + \frac{\alpha_s}{4\pi} D_{ij} O_j^L(a^{-1}) , \qquad (17)$$

where the lattice (continuum) operators are labelled by the superfix L (C). The mixing of operators is such that it is necessary to compute the matrix elements of eight lattice operators $O_j^L(a^{-1})$ (see table 1, where the coefficients D_{ij} are presented).

ii) We then evolve the HQET operators in the continuum scheme from renormalization scale $\mu = a^{-1}$ to scale $\mu = m_b$,

$$O_i^C(m_b) = M_{ij}(a^{-1}, m_b) O_j^C(a^{-1}) . \qquad (18)$$

This evolution, sometimes known as "hybrid renormalization", requires knowledge of the anomalous dimension matrix [16, 17, 18].

The matching procedure involves short-distance physics only, and so can be carried out in perturbation theory. Lattice perturbation theory generally

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[1]From these it is possible to obtain the matrix elements in any other renormalization scheme using perturbation theory.

D_{ij}	$i=1$	$i=2$	$i=3$	$i=4$	O_j^L
$j=1$	-21.64	-	2.06	0.54	$\bar{b}_L\gamma^\mu q_L\,\bar{q}_L\gamma^\mu b_L$
$j=2$	-	-21.64	2.16	2.40	$\bar{b}_R q_L\,\bar{q}_L b_R$
$j=3$	9.29	2.43	-10.80	2.83	$\bar{b}_L\gamma^\mu T^a q_L\,\bar{q}_L\gamma^\mu T^a b_L$
$j=4$	9.72	10.79	11.34	-9.05	$\bar{b}_R T^a q_L\,\bar{q}_L T^a b_R$
$j=5$	-18.37	-	-3.06	-	$\bar{b}_R\gamma^\mu q_R\,\bar{q}_L\gamma^\mu b_L$
$j=6$	36.75	18.37	6.12	3.06	$\bar{b}_R q_L\,\bar{q}_R b_L$
$j=7$	-13.78	-	6.89	-	$\bar{b}_R\gamma^\mu T^a q_R\,\bar{q}_L\gamma^\mu T^a b_L$
$j=8$	27.56	13.78	-13.78	-6.89	$\bar{b}_R T^a q_L\,\bar{q}_R T^a b_L$

Table 1: Coefficients D_{ij} for the four operators of eqs.(5)-(8).

converges very slowly, and therefore, where possible, the matching should be performed non-perturbatively so as to minimise these systematic errors. In this letter, however, we do not use a non-perturbative renormalization, but, when evaluating the matching coefficients, we do use a "boosted" lattice coupling constant in order to partially resum the large higher order contributions (e.g. those coming from tadpole graphs).

In order to obtain the parameters B_i and ε_i it is also necessary to determine the normalization of the axial current [19]. In this case there is a single coefficient Z_A defined by

$$\langle 0|\, A_0^C(a^{-1})\, |B\rangle = Z_A\, \langle 0|\, A_0^L(a^{-1})\, |B\rangle \ , \tag{19}$$

which at one loop order in perturbation theory is given by:

$$Z_A = 1 + \frac{\alpha_s(a^{-1})}{4\pi}\frac{4}{3}\left(\frac{5}{4} - \frac{1}{2}x_1 - x_2 + x_3\right) \simeq 1 - 20.0\,\frac{\alpha_s(a^{-1})}{4\pi} = 0.79 \tag{20}$$

where the coefficients x_i are defined in eq. (65) and tabulated in table 5 in Appendix A. A_0^C and A_0^L are the time components on the axial current in the HQET in the $\overline{MS}$ and lattice schemes respectively, both defined at the scale a^{-1}. Using hybrid renormalization one can obtain the axial current in the $\overline{MS}$ scheme at scale m_b. We stress again that the results presented for the B_i's and ε_i's below were obtained with both the four quark operators and

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the axial current defined in the HQET in the $\overline{MS}$ scheme at the scale m_b [2].

In the following section we combine the results of the matrix elements computed on the lattice with the perturbative coefficients presented in this section to obtain the B_i's and ε_i's.

3   Lattice Computation and Results

The non-perturbative strong interaction effects in spectator contributions to inclusive decays are contained in the matrix elements of the eight four-quark operators, O_j, given in table 1. We evaluate these matrix elements in a quenched simulation on a $24^3 \times 48$ lattice at $\beta = 6.2$ using the SW tree-level improved action [15]:

$$S^{SW} = S^{\text{gauge}} + S^{\text{Wilson}} - i\frac{\kappa}{2} \sum_{x,\mu,\nu} \bar{q}(x) F_{\mu\nu}(x) \sigma_{\mu\nu} \, q(x) \ , \tag{21}$$

where S^{gauge} and S^{Wilson} are the Wilson gauge and quark actions respectively. The use of this action reduces the errors due to the granularity of the lattice to ones of $O(\alpha_s a)$, where a is the lattice spacing. We use the 60 SU(3) gauge-field configurations, and the light quark propagators corresponding to hopping parameters $\kappa = 0.14144$, 0.14226 and 0.14262 which have been used previously to obtain the B-parameter of B^0-$\bar{B}^0$ mixing [12] and other quantities required for studies of B-physics. The value of the hopping parameter in the chiral limit is given by $\kappa_c = 0.14315$. The calculation of the matrix elements of the operators in table 1 is very similar to that of the $\Delta B = 2$ operators from which the B-parameter is extracted, and we exploit the similarities to perform checks on our calculations (see section 4 and Appendix B).

The evaluation of the matrix elements requires the computation of two- and three-point correlation functions:

$$C(t_x) \equiv \sum_{\vec{x}} \langle\, 0 \,|\, J(x) J^\dagger(0) \,|\, 0 \,\rangle \ , \tag{22}$$

where we have assumed that $t_x > 0$, and

$$K(t_x, t_y) \equiv \sum_{\vec{x}, \vec{y}} \langle\, 0 \,|\, J(y) O_j^L(0) J^\dagger(x) \,|\, 0 \,\rangle \ , \tag{23}$$

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[2]These differ at one-loop level from the corresponding operators in QCD.

where $t_y > 0 > t_x$. In eqs. (22) and (23) $J^\dagger$ and J are interpolating operators which can create or destroy a heavy pseudoscalar meson (containing a static heavy quark) and O_j is one of the operators whose matrix element we wish to evaluate. In practice we choose J to be the fourth component of an axial current. It is generally advantageous to "smear" the interpolating operators J and $J^\dagger$ over the spatial coordinates, in order to enhance the overlap with the ground state. Following ref. [12] we have used several methods of smearing and checked that the results for the matrix elements of O_j are independent of the choice of smearing. In this paper we present as our "best" results those obtained using a gauge-invariant smearing based on the Jacobi algorithm described in ref. [20]. The smeared heavy-quark field at time t, $b^S(\vec{x}, t)$, is defined by

$$b^S(x) = \sum_{\vec{x}'} M(\vec{x}, \vec{x}')b(\vec{x}', t) , \tag{24}$$

where

$$M(\vec{x}, \vec{x}') = \sum_{n=0}^{N} \kappa_S \Delta^n(\vec{x}, \vec{x}') \tag{25}$$

and Δ is the three-dimensional version of the Laplace operator. The parameter κ_S and the number of iterations can be used to control the smearing radius, and here we present our results for $\kappa_s = 0.25$ and $N = 140$ which corresponds to an rms smearing radius $r_0 = 6.4a$ [21]. The smeared interpolating operator is then chosen to be $J^S(x) = \bar{b}^S(x)\gamma_5 q(x)$, where q is the field of the light quark. The local interpolating operator is simply defined to be $J^L(x) = \bar{b}(x)\gamma_5 q(x)$.

The eight matrix elements $\langle B| O_j^L(0) |B\rangle$ are determined by computing the ratios:

$$R_j(t_1, t_2) \equiv \frac{4K_j^{SS}(-t_1, t_2)}{C^{LS}(-t_1)C^{LS}(t_2)} , \tag{26}$$

where t_1 and t_2 are positive and sufficiently large to ensure that only the ground state contributes to the correlation functions. The indices S and L denote whether the interpolating operators are smeared or local. It is convenient to choose both J and $J^\dagger$ to be smeared in the three-point function

K_j and to evaluate the two-point functions with a local operator at the source and a smeared one at the sink. At large time separations t_1 and t_2,

$$R_j(t_1, t_2) \rightarrow \frac{2}{m_B Z_L^2} \langle B| O_j^L(0) |B\rangle \ , \tag{27}$$

where Z_L is given by

$$\langle 0| J^L(0) |B\rangle \equiv \sqrt{2m_B} Z_L \ . \tag{28}$$

The R_j defined in eq.(26) contribute directly to the B_i's and the ε_i's, apart from perturbative matching factors. To see this, note that the leptonic decay constant of the B-meson in the static limit (i.e. infinite mass limit for the b-quark) is given by

$$f_B^2 = \frac{2Z_L^2 Z_A^2}{m_B} \ , \tag{29}$$

so that

$$R_j(t_1, t_2) \rightarrow \frac{2}{m_B Z_L^2} \langle B| O_j^L(0) |B\rangle = \frac{4Z_A^2}{f_B^2 m_B^2} \langle B| O_j^L(0) |B\rangle \ , \tag{30}$$

which corresponds precisely to the normalization of the B_i's and ε_i's in eqs.(9)–(12) (apart from the matching factors Z_A^2 and that of the four-quark operator O_j described in section 2).

In these computations it is particularly important to establish that the contribution from the ground-state has been isolated. The natural way to do this is to look for plateaus, i.e. for regions in t_1 and t_2 for which $R_j(t_1, t_2)$ is independent of t_1 and t_2. Since the statistical errors grow fairly quickly with $t_{1,2}$ our ability to verify the existence of plateaus is limited. For example in fig. 1 we present our results for $R_j(t_1, t_2)$, obtained with the light quark mass corresponding to $\kappa = 0.14226$ as a function of t_2 for $t_1 = 3$. The results are consistent with being constant, but the errors are uncomfortably large for $t_2 > 5$ or so. An alternative, and perhaps more convincing, way to confirm that the contribution from the ground-state has been isolated is to check that the values of the R_j are independent of the method of smearing used to define the smeared interpolating operators. Following ref. [12], in addition to the gauge invariant prescription for smearing described above, we have defined the smeared field in four other ways (having transformed the fields

	Gauge Inv.	Exponential	Gaussian	Cube	Double Cube
$R_1(3,3)$	1.02(3)	1.03(2)	1.04(3)	1.03(2)	1.07(8)
$R_2(3,3)$	1.00(2)	1.01(2)	1.01(3)	1.01(2)	1.00(5)
$R_3(3,3)$	0.00(1)	$-0.00(1)$	$-0.00(2)$	$-0.00(1)$	$-0.03(3)$
$R_4(3,3)$	0.00(1)	0.00(1)	0.01(1)	0.00(1)	0.01(2)
$R_5(3,3)$	$-0.99(4)$	$-1.01(3)$	$-1.01(4)$	$-1.00(3)$	$-1.01(7)$
$R_6(3,3)$	$-0.99(3)$	$-1.00(2)$	$-1.01(3)$	$-1.00(3)$	$-1.01(6)$
$R_7(3,3)$	$-0.04(2)$	$-0.03(2)$	$-0.04(2)$	$-0.03(2)$	$-0.09(5)$
$R_8(3,3)$	$-0.00(1)$	$-0.00(1)$	$-0.01(1)$	$-0.01(1)$	$-0.02(2)$

Table 2: Values of the $R_j(3,3)$ obtained with different smearing methods for a light quark with $\kappa = 0.14226$.

to the Coulomb gauge). For these additional four cases, $M(\vec{x}, \vec{x}')$ of eq. (25) is replaced by the following:

$$\text{Exponential}: \qquad M(\vec{x}, \vec{x}') = \exp(-|\vec{x} - \vec{x}'|/r_0), \tag{31}$$

$$\text{Gaussian}: \qquad M(\vec{x}, \vec{x}') = \exp(-|\vec{x} - \vec{x}'|^2/r_0^2), \tag{32}$$

$$\text{Cube}: \qquad M(\vec{x}, \vec{x}') = \prod_{i=1}^{3} \Theta(r_0 - |\vec{x}_i - x_i'|), \tag{33}$$

$$\text{Double Cube}: \qquad M(\vec{x}, \vec{x}') = \prod_{i=1}^{3} \left(1 - \frac{|x_i - x_i'|}{2r_0}\right) \Theta(2r_0 - |x_i - x_i'|), \tag{34}$$

and we have chosen $r_0 = 5$. As an example, we present in table 2 the results for the $R_j(t_1, t_2)$ for $t_1 = 3$ and $t_2 = 3$, again for the middle value of the three κ's, $\kappa = 0.14226$.

We take as our best results those obtained with the gauge invariant smearing, and with $t_1 = t_2 = 3$. After extrapolating to the chiral limit we find the following results for the R_j:

$$R_1(3,3) = 1.04 \pm 0.04 \qquad R_2(3,3) = 1.00 \pm 0.03 \tag{35}$$

$$R_3(3,3) = -0.01 \pm 0.02 \qquad R_4(3,3) = -0.00 \pm 0.01 \tag{36}$$

$$R_5(3,3) = -0.97 \pm 0.06 \qquad R_6(3,3) = -0.98 \pm 0.05 \tag{37}$$

$$R_7(3,3) = -0.03 \pm 0.03 \qquad R_8(3,3) = -0.01 \pm 0.01 \tag{38}$$

We now combine these results with the matching coefficients presented in section 2 to determine the B_i's and the ε_i's. For the value of the lattice
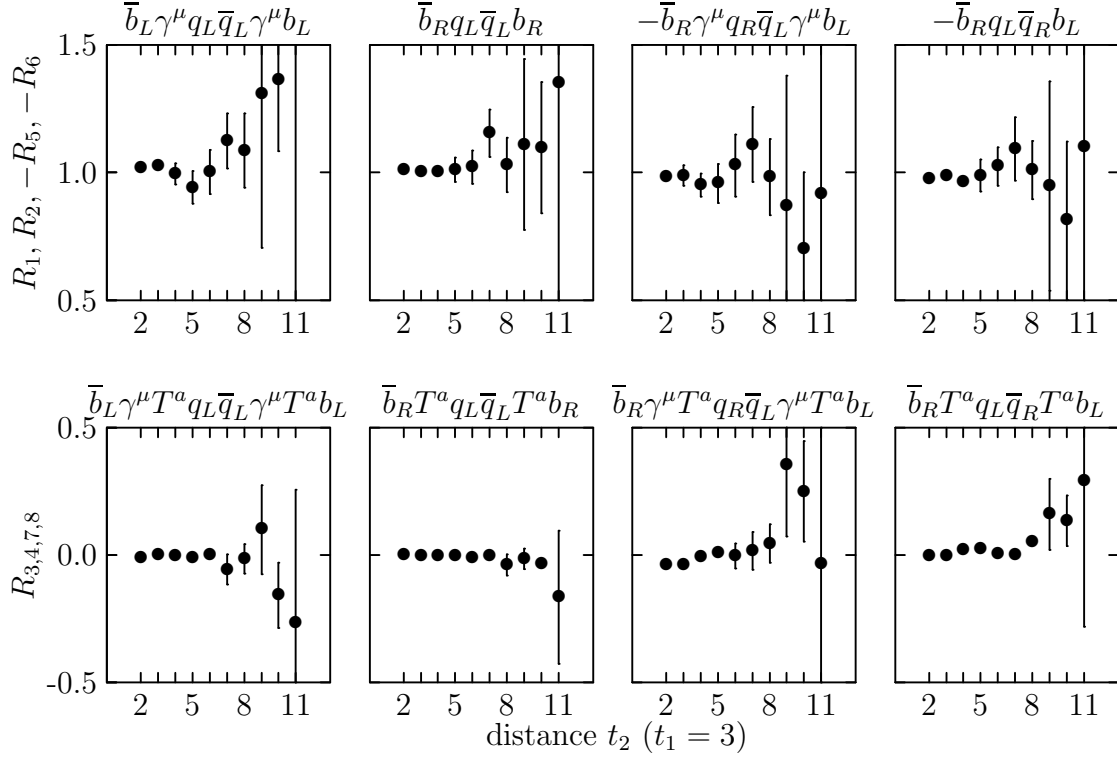
Figure 1: Results for the R_j, $j = 1 \cdots 8$ as a function of t_2 for $t_1 = 3$. The value of the quark mass corresponds to $\kappa = 0.14226$.

coupling constant we take one of the standard definitions of the boosted coupling:

$$\frac{\alpha_s(a^{-1})}{4\pi} = \frac{6(8\kappa_c)^4}{(4\pi)^2\beta} = 0.0105 \ . \tag{39}$$

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Using the coefficients from section 2 we then obtain

$$B_1(a^{-1}) = (Z_A)^{-2} \left(1 + \frac{\alpha_s(a^{-1})}{4\pi} D\right)_{1j} R_j = 0.98(8)$$

$$B_2(a^{-1}) = (Z_A)^{-2} \left(1 + \frac{\alpha_s(a^{-1})}{4\pi} D\right)_{2j} R_j = 0.93(5)$$

$$\varepsilon_1(a^{-1}) = (Z_A)^{-2} \left(1 + \frac{\alpha_s(a^{-1})}{4\pi} D\right)_{3j} R_j = 0.01(3)$$

$$\varepsilon_2(a^{-1}) = (Z_A)^{-2} \left(1 + \frac{\alpha_s(a^{-1})}{4\pi} D\right)_{4j} R_j = 0.00(2)$$

We now evolve those coefficients to the scale m_b, using the relations:

$$B(m_b) = \left[1 + \frac{2C_F\delta}{N_c}\right] B(a^{-1}) - \frac{2\delta}{N_c}\varepsilon(a^{-1}) \qquad (40)$$

$$\varepsilon(m_b) = \left[1 + \frac{\delta}{N_c^2}\right] \varepsilon(a^{-1}) - \frac{C_F\delta}{N_c^2} B(a^{-1}) , \qquad (41)$$

where

$$\delta \equiv \left(\frac{\alpha_s(a^{-1})}{\alpha_s(m_b)}\right)^{9/2\beta_0} - 1 = 0.09(3) . \qquad (42)$$

In estimating δ and its uncertainty we have allowed for a conservative variation of the parameters around the "central" values ($\Lambda_{QCD} = 250\,\text{MeV}$, $a^{-1} = 2.9\,\text{GeV}$, $m_b = 4.5\,\text{GeV}$ and $\beta_0 = 9$). We finally obtain:

$$B_1(m_b) = 1.06(8) \qquad B_2(m_b) = 1.01(6) \qquad (43)$$
$$\varepsilon_1(m_b) = -0.01(3) \qquad \varepsilon_2(m_b) = -0.01(2) . \qquad (44)$$

These matrix elements have also been evaluated using QCD sum-rules [22]. These authors find $B_1(m_b) = 0.96(4)$, $B_2(m_b) = 0.95(2)$, $\varepsilon_1(m_b) = -0.14(1)$ and $\varepsilon_2(m_b) = -0.08(1)$, differing somewhat (particularly for the ε_i's) from our values.

Using the results in eqs. (43) and (44) we obtain the following value for the ratio of lifetimes for the neutral and charged B-mesons:

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + k_1 B_1 + k_2 B_2 + k_3 \varepsilon_1 + k_4 \varepsilon_2 = 1.03 \pm 0.02 \pm 0.03 \qquad (45)$$

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where the coefficients k_i are taken from ref. [9]. The first error in eq. (45) is from the uncertainty in the values of the matrix elements in eqs. (43) and (44), whereas the second is an estimate of the uncertainty due to our ignorance of the one-loop contribution to the Wilson coefficient function in the OPE (this was estimated by varying the matching scale from $m_b/2$ to $2m_b$ using the procedure described in ref. [9]). The value in eq. (45) is in good agreement with the experimental results in eq. (3).

There has been a considerable amount of discussion lately as to whether the theoretical predictions for the semileptonic branching ratio of the B-meson (B_{SL}) and the charm yield are consistent with experimental measurements [23, 24, 9]. Here we limit our discussion to a comment on the implications of our results on these two physical quantities. The spectator contributions to B_{SL} and n_c take the form:

$$\Delta B_{SL,\text{spec}} = b_1 B_1 + b_2 B_2 + b_3 \varepsilon_1 + b_4 \varepsilon_2 \qquad (46)$$

$$\Delta n_{c,\text{spec}} = n_1 B_1 + n_2 B_2 + n_3 \varepsilon_1 + n_4 \varepsilon_2 \ , \qquad (47)$$

where the coefficients b_i and n_i at tree-level are presented explicitly in ref. [9] (see also refs. [11, 25, 16]). The uncertainties in the predictions for B_{SL} and n_c due to the errors in the matrix elements in eqs. (43) and (44) are small (about 0.1% for both B_{SL} and n_c). Larger uncertainties are due to our ignorance of the one-loop contribution to the Wilson coefficient function in the OPE. For example, following the procedure described in ref. [9] with a matching scale $\mu = m_b/2$ we find $\Delta B_{SL,\text{spec}} = 0.3(1)\%$ and $\Delta n_{c,\text{spec}} = 0.5(2)\%$, whereas for a scale $2m_B$ the corresponding results are $\Delta B_{SL,\text{spec}} = -0.1(1)\%$ and $\Delta n_{c,\text{spec}} = 0.0(1)\%$. For the charm yield these corrections are negligible as expected, and for the semileptonic branching ratio they are small. Nevertheless, it would be useful to know the one-loop contribution to the coefficient functions in order to eliminate the variation obtained by changing the matching scale.

4 B-$\bar{B}$ Mixing

In this section we revisit the phenomenologically important process of B-$\bar{B}$ mixing. The computation of the matrix elements of the relevant lattice operators on the same gauge configurations has already been presented in ref. [12], and we do not add to these lattice computations. We do, however, wish to make two observations:

i) The matrix elements of the lattice $\Delta B = 2$ operators relevant for B-$\bar{B}$ mixing factorize with a similar precision to that found for the four-quark operators considered in section 3. Because of the way in which lattice computations are usually organised, this property is not as readily manifest for the ΔB=2 operators as it is for the spectator effects. However, by using colour and spin Fierz identities, we demonstrate that the values of the matrix elements of the lattice $\Delta B = 2$ operators are very close to those expected using factorization. Some of our observations have already been noted in ref. [14], where the Wilson formulation for the light quarks was used. We extend this investigation of factorization, and show that each contribution to the matrix elements (i.e. each Wick contraction) is close to the estimated value obtained using the factorization and vacuum saturation hypothesis.

ii) We believe that there is an error in the published value of the matching factors for the $\Delta B = 2$ operators using the SW action for the light quarks. We discuss this in some detail in Appendix B; in this section we briefly comment on the consequences of this error.

We now consider these two observations in turn.

4.1   Factorization

In order to determine the B-parameter of B-$\bar{B}$ mixing we need to evaluate the matrix elements of the three lattice operators O_L, O_R and O_N defined in eqs.(66)–(68) of Appendix B, as well as that of

$$O_S = \bar{b}q_L\,\bar{b}q_L \ . \tag{48}$$

We now show that these matrix elements (and related ones) are reproduced remarkably accurately by assuming the factorization hypothesis and vacuum saturation. For each of these operators (O_j) we define the ratio $R_j(t_1, t_2)$ analogously to eq.(26) as follows:

$$R_j(t_1, t_2) \equiv \frac{3}{8}\frac{K_j^{SS}(t_1, t_2)}{C^{LS}(-t_1)C^{LS}(t_2)} \ , \tag{49}$$

where the correlation functions C and K_j are defined in eqs.(22) and (23), except that in the K_j the operators O_j are now $\Delta B = 2$ operators, and the interpolating operators destroy a B-meson and create a $\bar{B}$-meson.

We start by explaining explicitly what we mean by factorization (combined with vacuum saturation). The evaluation of the B-parameter requires the computation of three-point functions $K_j(t_1, t_2)$, which are of the form [3]:

$$\langle\, 0\, |\bar{q}(y)\gamma^5 b(y)\,\bar{b}(0)\Gamma q(0)\,\bar{b}(0)\tilde{\Gamma}q(0)\,\bar{q}(x)\gamma^5 b(x)\,|\, 0\,\rangle\;. \tag{50}$$

There are four Wick contractions which contribute to the correlation function, and it is convenient to track these by introducing a fictitious quantum number, labelled by an integer suffix on each field, so that the correlation function is written in the form

$$\langle\, 0\, |\bar{q}_2(y)\gamma^5 b_1(y)\, O_{\Gamma,\tilde{\Gamma}}(0)\,\bar{q}_4(x)\gamma^5 b_3(x)\,|\, 0\,\rangle\;, \tag{51}$$

where

$$O_{\Gamma\tilde{\Gamma}} \equiv \bar{b}_1\Gamma q_2\,\bar{b}_3\tilde{\Gamma}q_4\; +\bar{b}_3\Gamma q_4\,\bar{b}_1\tilde{\Gamma}q_2\; -\bar{b}_1\Gamma q_4\,\bar{b}_3\tilde{\Gamma}q_2\; -\bar{b}_3\Gamma q_2\,\bar{b}_1\tilde{\Gamma}q_4\;, \tag{52}$$

and $\Gamma,\tilde{\Gamma}$ are Dirac matrices. The contraction of spinor (α,β) and colour (a) indices in each bilinear in eq. (52) is implicit (e.g. $\bar{b}_1\Gamma q_2 = \bar{b}_{1,\alpha}^a\Gamma_{\alpha,\beta}q_{2,\beta}^a$). Only contractions between fields with the same suffix are allowed, and the four terms in eq. (52) correspond to the four original Wick contractions. For all the operators of interest, by using colour and spin Fierz identities, it is possible to rewrite each of the four terms into sums of operators of the form $(\bar{b}_1\Gamma'q_2)\,(\bar{b}_3\tilde{\Gamma}'q_4)$ and $(\bar{b}_1\Gamma'T^a q_2)\,(\bar{b}_3\tilde{\Gamma}'T^a q_4)$, for some γ-matrices Γ' and $\tilde{\Gamma}'$. In the factorization and vacuum saturation hypothesis

$$\langle\,\bar{B}\,|(\bar{b}_1\Gamma'T^a q_2)\,(\bar{b}_3\tilde{\Gamma}'T^a q_4)\,|\,B\,\rangle\simeq 0\;, \tag{53}$$

and

$$\langle\,\bar{B}\,|(\bar{b}_1\Gamma'q_2)\,(\bar{b}_3\tilde{\Gamma}'q_4)\,|\,B\,\rangle\simeq\langle\,\bar{B}\,|(\bar{b}_1\Gamma'q_2)\,|\,0\rangle\langle\,0\,|\,(\bar{b}_3\tilde{\Gamma}'q_4)\,|\,B\,\rangle\;. \tag{54}$$

Lorentz invariance implies that each of the matrix elements on the right-hand side of eq.(54) vanishes or is proportional to the leptonic decay constant f_B.

We now consider each of the operators in turn:

- O_L and O_R: The factor of 3/8 in eq.(49) was chosen so that the factorization hypothesis gives $R_L = R_R = 1$. Numerically we find:

$$\frac{R_L(3,3) + R_R(3,3)}{2} = 0.95 \pm 0.03\;. \tag{55}$$

---

[3]The extension of this discussion to include smeared interpolating operators is completely straightforward.

- O_S: Lorentz invariance implies that the matrix element $\langle B | \bar{b}\sigma^{\mu\nu}(1 - \gamma^5)q | 0 \rangle$ vanishes. Using this fact we deduce that factorization implies that $R_S \simeq 5/8$. The numerical result for R_S is $0.60(3)$, in very good agreement with the estimate based on factorization.

- O_N: Finally factorization implies that $R_N \simeq 1$, in good agreement with the numerical value $0.97(4)$.

In order to illustrate further that the numerical results quoted above are in agreement with expectations based on factorization and vacuum saturation we present separately in fig. 2 the contributions to each of the ratios from the two independent contractions:

$$R^a = \langle \bar{B} | (\bar{b}_1 \Gamma' q_2)(\bar{b}_3 \tilde{\Gamma}' q_4) | B \rangle \tag{56}$$

and

$$R^b = \langle \bar{B} | (\bar{b}_1 \Gamma' q_4)(\bar{b}_3 \tilde{\Gamma}' q_2) | B \rangle \ , \tag{57}$$

as well as the total combination

$$R = \frac{3}{8}(2R^a - 2R^b) \tag{58}$$

We see that not only are the values of the R's quoted above in agreement with expectations from factorization, but the values of each of the R^a and R^b are also those we would expect on this assumption:

$$R_L^a \simeq 1 \qquad R_L^b \simeq -\frac{1}{3}R_L^a \qquad ; \qquad R_S^a \simeq -1 \qquad R_S^b \simeq \frac{1}{6}R_S^a \tag{59}$$

$$R_R^a \simeq 1 \qquad R_R^b \simeq -\frac{1}{3}R_L^a \qquad ; \qquad R_N^a \simeq 2 \qquad R_N^b \simeq \frac{1}{3}R_N^a \ . \tag{60}$$

In this subsection we have demonstrated that the surprising precision of predictions for matrix elements obtained using the factorization hypothesis in inclusive decays, which was discussed in section 3, also applies to existing results for B-$\bar{B}$ mixing.

4.2 Matching

In this subsection we discuss very briefly the implications of the error in the published value of the perturbative matching factors in B-$\bar{B}$ mixing (see
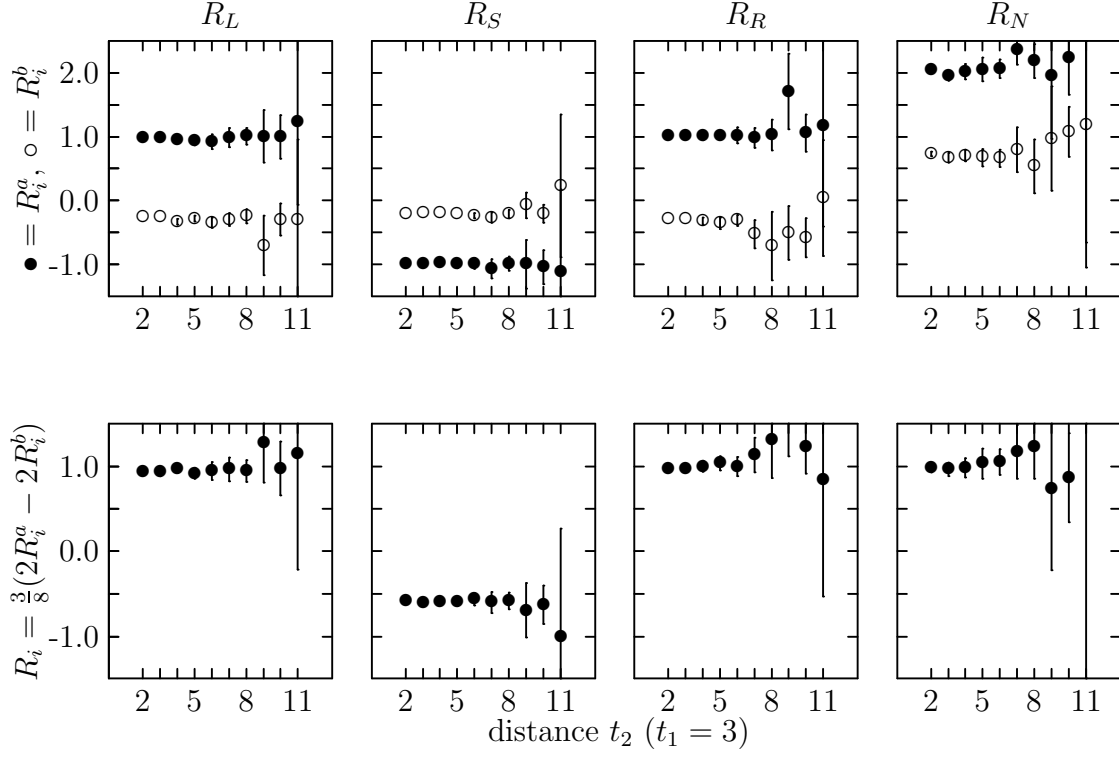
Figure 2: Results for the R_j^a, R_j^b and R_j ($j = L, S, R, N$) as a function of t_2 for $t_1 = 3$. The value of the quark mass corresponds to $\kappa = 0.14226$.

appendix B). For example, we take method (a) of ref. [12], in which each of the ratios R_i is fitted separately, and find

$$B_b(m_b) = 0.66(2) \ , \qquad B_b = \alpha_s^{-6/23} B_b(m_b) = 0.96(3) \ , \qquad (61)$$

where $B_b(m_b)$ and B_b are the B-parameter in the $\overline{\text{MS}}$ scheme at scale m_b and the renormalization group invariant B-parameter respectively. For the comparison we only quote the statistical error. In order to see the effect of the error in the matching coefficient, the result in eq. (61) should be compared with that obtained in ref. [12] using an identical procedure (except for the values of the matching coefficients):

$$B_b(m_b) = 0.69(2) \ , \qquad B_b = \alpha_s^{-6/23} B_b(m_b) = 1.02(3) \ . \qquad (62)$$

Giménez and Martinelli had pointed out that the authors of ref. [12] had used matching coefficient which did not include the contributions to the

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$O(a^2)$ terms in the lattice operators which they had used [13]. As explained in ref. [13], this leads to a negligible correction to the B-parameter. The differences in the values in eqs. (61) and (62) is therefore largely due to the error discussed in appendix B.

5 Conclusions

In this paper we have evaluated the matrix elements which contain the non-perturbative QCD effects in the spectator contributions to inclusive decays of B-mesons. For the $\overline{MS}$ scheme our principal results are contained in eqs. (43) and (44). The raw lattice results from which the matrix elements in any renormalization scheme can be determined are given in eqs.(35)–(38).

The results for the matrix elements are very close to those which would be expected on the basis of factorization and the vacuum saturation hypothesis (particularly for the matrix elements of the bare lattice operators). We find the extent to which this hypothesis is satisfied to be surprising, but point out in subsec. 4.1 that this was also the case for the $\Delta B = 2$ operators which contribute to B-$\bar{B}$ mixing, whose matrix elements have been computed by several groups.

The calculation described in this paper is the first evaluation, using lattice simulations, of the matrix elements of the operators in eqs.(5)–(8) between B-meson states. The errors in the results presented in eqs.(35)–(38) are reasonably small, and until unquenched computations become possible, it is not phenomenologically necessary to invest a large effort to reduce the statistical errors in this quenched calculation. It is desirable, however, to establish, beyond any doubt, that the ground state meson has been isolated and to confirm explicitly the validity of the arguments presented in sec. 3 that this is indeed the case. For this a similar calculation on a larger statistical sample will be required. It would also be very useful to evaluate the one-loop contributions to the Wilson coefficient functions in the OPE for inclusive decay rates. It is probable that these corrections currently represent the largest uncertainty in the inclusive rates.

A similar study of the matrix elements of the operators eqs.(35)–(38) between Λ_b states is in progress. This is particularly important in view of the discrepancy between the experimental measurement in eq. (1) and the theoretical prediction in eq. (2) for the ratio $\tau(\Lambda_b)/\tau(B_d)$. This calculation will help determine whether the discrepancy is due to baryonic matrix ele-

ments being larger than expected from quark models, or whether local quark-hadron duality, assumed in the phenomenology, is not valid. Technically the baryonic matrix elements of four-quark operators are more complicated to evaluate than mesonic ones, since it is not sufficient to generate the set of light-quark propagators from an arbitrary lattice point to the origin. The results will be presented in a following publication.

Appendix A - Evaluation of the D_{ij}

In this appendix we discuss the evaluation of the coefficients $\{D_{ij}\}$ of eq. (17) for a generic four-quark operator O_i. This requires the evaluation of one loop corrections to the matrix elements of the operators $\{O_i\}$ in both the continuum and lattice schemes:

$$D_{ij} = C_{ij}^C - C_{ij}^L \ , \tag{63}$$

where the superscripts C and L refer to the continuum renormalization scheme ($\overline{MS}$) and the lattice regularization respectively. We evaluate the matrix elements between four-quark states at zero momentum, regulating the infrared divergences by giving the gluon a small mass λ. The coefficients $\{D_{ij}\}$ do not depend on the infra-red regulator, although each of C_{ij}^C and C_{ij}^L are separately infrared divergent.

We distinguish five categories of one-loop corrections to the generic operator

$$O_i \equiv \bar{b}\Gamma_i^A q \, \bar{q}\tilde{\Gamma}_i^A b \ , \tag{64}$$

where A represents both colour and spinor indices:

1. Corrections to the external lines. These are proportional to C_F, the eigenvalue of the quadratic Casimir operator in the fundamental representation ($C_F = 4/3$ for the $SU(3)$ colour group), and are independent of Γ_i and $\tilde{\Gamma}_i$.

2. Vertex corrections to each of the quark bilinears.

3. Corrections in which the gluon couples to both heavy-quark propagators.

(category)	C_{ij}^C	O_j^L
(1)	$-\log(\lambda^2 a^2) + \frac{1}{2}$	$C_F\, \bar{b}\Gamma_i^A q\, \bar{q}\tilde{\Gamma}_i^A b$
(2)	$-\log(\lambda^2 a^2) + 1$	$\bar{b}(T^a\Gamma_i^A T^a)q\, \bar{q}\tilde{\Gamma}_i^A b + \bar{b}\Gamma_i^A q\, \bar{q}(T^a\tilde{\Gamma}_i^A T^a)b$
(3, odd)	$2\log(\lambda^2 a^2)$	$\bar{b}(T^a\Gamma_i^A)q\, \bar{q}(\tilde{\Gamma}_i^A T^a)b$
(4, odd)	$\log(\lambda^2 a^2) - 1$	$\bar{b}(\Gamma_i^A T^a)q\, \bar{q}(\tilde{\Gamma}_i^A T^a)b + \bar{b}(T^a\Gamma_i^A)q\, \bar{q}(T^a\tilde{\Gamma}_i^A)b$
(5, odd)	$-\log(\lambda^2 a^2) + \Omega_1$	$\bar{b}(\Gamma_i^A T^a)q\, \bar{q}(T^a\tilde{\Gamma}_i^A)b$
(5, odd)	$-3\log(\lambda^2 a^2) + \Omega_2$	$\frac{1}{12}\bar{b}(\Gamma_i^A \sigma^{\mu\nu} T^a)q\, \bar{q}(T^a\sigma^{\nu\mu}\tilde{\Gamma}_i^A)b$

Table 3: Values of the contributions to C_{ij}^C.

4. Corrections in which the gluon couples to one light-quark propagator and one heavy-quark propagator (at the other vertex).

5. Corrections in which the gluon couples to both light-quark propagators.

In table 3 we present the values of the contributions to the C_{ij}^C in the $\overline{MS}$ scheme. For the operators O_1 and O_3, $\Omega_1 = -\frac{1}{2}$ and $\Omega_2 = 0$, whereas for O_2 and O_4 $\Omega_1 = 1$ and $\Omega_2 = 0$. For a general choice of O_i, $\Omega_{1,2}$ depend on the precise definition of the operator $\bar{b}(\Gamma_i^A \sigma^{\mu\nu})q\, \bar{q}(\sigma^{\nu\mu}\tilde{\Gamma}_i^A)b$ and on the choice of basis for the γ-matrices in D-dimensions.

The coefficients $\{D_{ij}\}$ themselves are presented in table 4, where the x_i's are defined as:

$$x_i \equiv c_i + c_i^I , \tag{65}$$

and the numerical values of the c_i and c_i^I are tabulated in table 5. The contributions proportional to the x_i's come from the diagrams in the lattice regularization. The components proportional to the c_i would be the results if the Wilson formulation of the quark action had been used, and those proportional to c_i^I are the additional contributions which result from the use of the SW-improved action [4].

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[4] In the notation of ref. [19], the parameters δ_v and δ_s are defined by

$$\delta_v \equiv \frac{1}{\pi^2} \int_{-\pi}^{+\pi} d^4 k \left(\frac{\Delta_4(4 - \Delta_1) - (\Delta_4 - \Delta_5)}{4\Delta_1\Delta_2^2} - 3\frac{\theta(1 - k^2)}{k^4} \right) \quad \text{and}$$

j (Category)	D_{ij}	O_j^L
1 (1,2)	$\frac{1}{2}-x_1$	$C_F\,\bar{b}\Gamma_i^A q\,\bar{q}\tilde{\Gamma}_i^A b$
2 (2)	$1-x_2$	$\bar{b}(T^a\Gamma_i^A T^a)q\,\bar{q}\tilde{\Gamma}_i^A b + \bar{b}\Gamma_i^A q\,\bar{q}(T^a\tilde{\Gamma}_i^A T^a)b$
3 (2)	$-x_3$	$\bar{b}(T^a\gamma^0\Gamma_i^A\gamma^0 T^a)q\,\bar{q}\tilde{\Gamma}_i^A b + \bar{b}\Gamma_i^A q\,\bar{q}(T^a\gamma^0\tilde{\Gamma}_i^A\gamma^0 T^a)b$
4 (3, odd)	$-x_4$	$\bar{b}(T^a\Gamma_i^A)q\,\bar{q}(\tilde{\Gamma}_i^A T^a)b$
5 (4, odd)	$-1-x_5$	$\bar{b}(\Gamma_i^A T^a)q\,\bar{q}(\tilde{\Gamma}_i^A T^a)b + \bar{b}(T^a\Gamma_i^A)q\,\bar{q}(T^a\tilde{\Gamma}_i^A)b$
6 (4)	$-x_6$	$\bar{b}(\Gamma_i^A\gamma^0 T^a)q\,\bar{q}(\tilde{\Gamma}_i^A\gamma^0 T^a)b + \bar{b}(T^a\gamma^0\Gamma_i^A)q\,\bar{q}(T^a\gamma^0\tilde{\Gamma}_i^A)b$
7 (5, odd)	Ω_1-x_7	$\bar{b}(\Gamma_i^A T^a)q\,\bar{q}(T^a\tilde{\Gamma}_i^A)b$
8 (5, odd)	Ω_2-x_8	$\frac{1}{12}\bar{b}(\Gamma_i^A\sigma^{\mu\nu}T^a)q\,\bar{q}(T^a\sigma^{\nu\mu}\tilde{\Gamma}_i^A)b$
9 (5)	$-x_9$	$\frac{1}{4}\bar{b}(\Gamma_i^A\gamma^\mu T^a)q\,\bar{q}(T^a\gamma^\mu\tilde{\Gamma}_i^A)b$

Table 4: Values of the contributions to D_{ij}.

Appendix B - $\Delta B = 2$ operators

There are many parallels between the calculations of spectator effects in inclusive decays, which is the main subject of this paper, and that of the matrix elements of the $\Delta B = 2$ operators which contribute, for example, to the important process of B^0-$\bar{B}^0$ mixing. Indeed, we have recomputed the matrix elements of the $\Delta B = 2$ operators and compared the results to those in ref. [12] as a check on our procedures and programs. The calculation of the matching factors are also similar in the two cases, and we have exploited this fact as a check on our perturbative calculation. Since we disagree with one of the terms in the results of ref. [19], we briefly discuss the evaluation of the matching factors in this appendix.

We consider a generic $\Delta B = 2$ operator of the form $\bar{b}\Gamma_i^A q\,\bar{b}\tilde{\Gamma}_i^A q$. The evaluation of the various contributions to the D_{ij}'s for $\Delta B = 2$ operators parallels that of the operator O_i for spectator effects in inclusive decays defined in eq. (64); specifically, as explicitly marked in table 4, the coefficients corresponding to $j = 4, 5, 7$ and 8 change sign, whilst the remaining coeffi-

$$\delta_s \equiv \frac{r^2}{4\pi^2}\int_{-\pi}^{+\pi} d^4k\,\frac{4\Delta_1(\Delta_4-\Delta_5)(2+r^2\Delta_1)-\Delta_4^2(2+r^2\Delta_1)}{4\Delta_1\Delta_2^2}\ .$$

$c_1 = f + e = 17.89$	$c_1^I = f^I - 2(l + m) = -11.53$
$c_2 = d_1 = 5.46$	$c_2^I = n = 0.73$
$c_3 = d_2 = -7.22$	$c_3^I = h - 2d^I - q = 0.33$
$c_4 = -c = -4.53$	$c_4^I = 0$
$c_5 = -d_1 = -5.46$	$c_5^I = -n = -0.73$
$c_6 = d_2 = -7.22$	$c_6^I = c_3^I = 0.33$
$c_7 = -v - \delta_v = 4.85$	$c_7^I = \delta_s = 0.27$
$c_8 = \delta_v = 2.07$	$c_8^I = -v^I + s - \delta_s + 3\delta_L = 10.46$
$c_9 = 4w = -4.84$	$c_9^I = 4w^I - 2l - 2m + s + 3\delta_R = -4.88$

Table 5: Values of the coefficients c_i and c_i^I. Their expressions are given in terms of the variables defined in refs. [19] and [26] and two new variables δ_v, δ_s. The numerical integrals have been recomputed. Some of them differ slightly from the results of [19].

cients are the same. Thus from table 4 for spectator effects we deduce that the D_{ij}'s for $\Delta B = 2$ operators are as given in table 6. The x_i's are defined in eq. (65) and tabulated in table 5.

We are particularly interested in the operator whose matrix elements contains the non-perturbative QCD effects for B^0-$\bar{B}^0$ mixing:

$$\bar{b}\Gamma_i^A q \, \bar{b}\widetilde{\Gamma}^A q \equiv O_L = \bar{b}\gamma^\rho q_L \, \bar{b}\gamma_\rho q_L \ , \tag{66}$$

where the label L denotes "left". In this case the different operators O_j of table 6 reduce to 3 independent ones:

$$O_R = \bar{b}\gamma^\rho q_R \, \bar{b}\gamma_\rho q_R \tag{67}$$
$$O_N = 2\bar{b}q_L \, \bar{b}q_R + 2\bar{b}q_R \, \bar{b}q_L + \bar{b}\gamma^\rho q_L \, \bar{b}\gamma_\rho q_R + \bar{b}\gamma^\rho q_R \, \bar{b}\gamma_\rho q_L \ , \tag{68}$$

as well as O_L itself. Step i) of the matching (see section 2) is now given by the relation:

$$O_L^C = (1 + \frac{\alpha_s}{4\pi} D_L) O_L^L + \frac{\alpha_s}{4\pi} D_N O_N^L + \frac{\alpha_s}{4\pi} D_R O_R^L \tag{69}$$

where

$$D_L = -22.4 \ , \ \ D_N = -13.8 \ \ \text{and} \ \ D_R = -3.2 \ . \tag{70}$$

j (category)	$D_{ij}^{\Delta B=2}$	O_j^L
1 (1,2)	$\frac{1}{2} - x_1$	$C_F\,\bar{b}(\Gamma_i^A)q\,\bar{b}(\widetilde{\Gamma_i^A})q$
2 (2)	$1 - x_2$	$\bar{b}(T^a\Gamma_i^A T^a)q\,\bar{b}(\widetilde{\Gamma_i^A})q + \bar{b}(\Gamma_i^A)q\,\bar{b}(T^a\widetilde{\Gamma_i^A}T^a)q$
3 (2)	$-x_3$	$\bar{b}(T^a\gamma^0\Gamma_i^A\gamma^0 T^a)q\,\bar{b}(\widetilde{\Gamma_i^A})q + \bar{b}(\Gamma_i^A)q\,\bar{b}(T^a\gamma^0\widetilde{\Gamma_i^A}\gamma^0 T^a)q$
4 (3, odd)	x_4	$\bar{b}(T^a\Gamma_i^A)q\,\bar{b}(T^a\widetilde{\Gamma_i^A})q$
5 (4, odd)	$1 + x_5$	$\bar{b}(\Gamma_i^A T^a)q\,\bar{b}(T^a\widetilde{\Gamma_i^A})q + \bar{b}(T^a\Gamma_i^A)q\,\bar{b}(\widetilde{\Gamma_i^A}T^a)q$
6 (4)	$-x_6$	$\bar{b}(\Gamma_i^A\gamma^0 T^a)q\,\bar{b}(T^a\gamma^0\widetilde{\Gamma_i^A})q + \bar{b}(T^a\gamma^0\Gamma_i^A)q\,\bar{b}(\widetilde{\Gamma_i^A}\gamma^0 T^a)q$
7 (5, odd)	$-\Omega_1 + x_7$	$\bar{b}(\Gamma_i^A T^a)q\,\bar{b}(\widetilde{\Gamma_i^A}T^a)q$
8 (5, odd)	$-\Omega_2 + x_8$	$\frac{1}{12}\bar{b}(\Gamma_i^A\sigma^{\mu\nu}T^a)q\,\bar{b}(\widetilde{\Gamma_i^A}\sigma^{\mu\nu}T^a)q$
9 (5)	$-x_9$	$\frac{1}{4}\bar{b}(\Gamma_i^A\gamma^\mu T^a)q\,\bar{b}(\widetilde{\Gamma_i^A}\gamma^\mu T^a)q$

Table 6: Values of the contributions to the D_{ij}'s for $\Delta B = 2$ operators.

For the continuum renormalization scheme we have used $\overline{MS}$. In $4 - 2\epsilon$ dimensions

$$\bar{b}\gamma^\rho\sigma^{\mu\nu}q_L\,\bar{b}\gamma_\rho\sigma_{\mu\nu}q_L = (12 - 2\epsilon)\bar{b}\gamma^\rho q_L\,\bar{b}\gamma_\rho q_L \ , \qquad (71)$$

from which we derive that $\Omega_1 + \Omega_2 = 5$ in this scheme. In considering the crossing relations between inclusive decays and mixing, we note that in the latter process, one of the $\bar{b}$ fields destroys a quark (so that $\bar{b}\gamma^0 = \bar{b}$) and the other creates an antiquark (so that $\bar{b}\gamma^0 = -\bar{b}$).

The results for D_L and D_N in eq. (70) agree with those in the literature [26], whereas that for D_R does not (a similar conclusion was reached independently by Giménez [27]). In ref. [26] the quoted result is $D_R = -5.4$ [5].

[5]We believe that the reason is that in eq. (B.16) of ref.[19] there should be a correction:

$$(\ldots) + \frac{g^2}{16\pi^2}\frac{4}{3}\left(\omega + \omega^I\right) \to (\ldots) - \frac{g^2}{16\pi^2}\frac{4}{3}\left(\omega + \omega^I\right)O_R^{latt} \ ,$$

eq. (B.26) should be replaced by

$$D_R^I = \frac{1}{3}\left[s + 4\omega^I - 2(l + m)\right]$$

and that in table 3

$$D_R^I = -0.38 \ \text{ for } r = 1 \ .$$

Acknowledgements

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There errors are carried forward into successive papers [26], [12] and [13]. Moreover in the same paper, in the definition of v^I (eq. (B.17)), the term Δ_2 at the denominator should be replaced by Δ_2^2.

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